Formulating the Null and Alternative Hypotheses

IMPORTANT: Hypotheses are \*always\* about population parameters. They are \*never\* about sample statistics, so if you write the notation for a sample statistic in a pair of hypotheses, that will be a signal you have done something wrong.

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First, let’s practice some basics, referring back to **Step #1: Formulating the Hypotheses** in Part 1 of *Ch9: Handout #2*.

*Example 1.* Write the hypotheses for a **lower tail test** for a single population mean if is .

*Example 2.* Write the hypotheses for an **upper tail test** for a single population mean if is

*Example 3.* Write the hypotheses for a **two-tailed test** for a single population mean if is

**Now, try it yourself:**

*Exercise 1.* Write the hypotheses for an **upper tail test** for a single population mean if is

*Exercise 2.* Write the hypotheses for a **lower tail test** for a single population mean if is .

*Exercise 3.* Write the hypotheses for a **two-tailed test** for a single population mean if is

Now, let’s build on that. To determine which tail test to choose, you have to match up the question you have about the population mean with the hypotheses that are the best fit to answer the question. Since you can only prove the Alternative Hypothesis (), and you cannot ever prove the null hypothesis (), it is usually helpful to frame your question in terms of the alternative hypothesis. What I mean by that is, you can read each aloud in words, and see which one fits the circumstances best.

*Example 4.* Formulate the hypotheses for a hypothesis test to determine whether a population mean is smaller than 14.

*Example 5.* Formulate the hypotheses for a hypothesis test to determine whether a population mean is different than 419.

*Example 6.* Formulate the hypotheses for a hypothesis test to determine whether a population mean has increased from 85.

**Now, try it yourself:**

*Exercise 4.* Formulate the hypotheses for a hypothesis test to determine whether a population mean has decreased from 581.

*Exercise 5.* Formulate the hypotheses for a hypothesis test to determine whether a population mean has changed from 74.

*Exercise 6.* Formulate the hypotheses for a hypothesis test to determine whether a population mean is higher than 620.

Now let’s apply what we’ve learned to actual hypothesis testing scenarios.

In order to properly formulate the Null Hypothesis () and the Alternative Hypothesis (), you must:

1. Identify the parameter of interest
2. Identify the question being asked about the parameter of interest and match it to an LT, UT, or 2T test

*Example 7:*

A shampoo company has introduced a new formula of a particular shampoo. In the past, customers have been regularly surveyed about the shampoo, asked to rate the product on a scale of 1 to 10, and the shampoo has scored a mean of 6.2.

An analyst at the company would like to test whether the mean satisfaction score changed after the new formula was introduced. Formulate the Null and Alternative Hypotheses the analyst should use.

1. The parameter of interest is the population mean customer satisfaction score
2. The question being asked is: has the mean customer satisfaction score changed from what it was before the new formula was introduced (which was 6.2)?

The hypotheses will refer to which is the population mean customer satisfaction score. The analyst wants to test for any change from 6.2 – if the mean score has changed, then it would be different from (i.e. not equal to) 6.2. Therefore, the proper hypotheses are:

*Example 8*:

The same situation as Example 1, except this time, the analyst is concerned that the mean customer satisfaction score has fallen with the introduction of the new formula.

Formulate the Null and Alternative Hypotheses to test whether the mean satisfaction score has decreased.

1. The parameter of interest is (still) the population mean customer satisfaction score
2. The question being asked is: has the mean customer satisfaction score decreased from what it was before (which was 6.2)?

The hypotheses will still refer to which is the mean customer satisfaction score of the population. The analyst wants to test for a decrease – and if the mean score has decreased than it would be less than 6.2. Therefore, the proper form is:

*Example 9:*

Again, same situation as Example 1. But this time, the analyst would like to prove that the new formula is better than the old formula. That is, the analyst would like to test whether the mean customer satisfaction score has increased.

Formulate the Null and Alternative Hypotheses to test whether the mean customer satisfaction score has gone up with the new formula’s introduction.

1. The parameter of interest is (still) the population mean customer satisfaction score
2. The question being asked is: Has the mean customer satisfaction score increased from what it was before (which was 6.2)?

The hypotheses will still refer to which is the mean customer satisfaction score of the population. The analyst would like to show that this parameter has increased – and if the mean score has increased than it would be greater than 6.2. Therefore, the hypotheses are:

*Example 10:*

A manufacturer makes test tubes for laboratory use. These test tubes must hold exactly 10 milliliters (ml) of liquid. A quality control analyst would like to check whether or not the production line is running properly. If the production line goes out of adjustment, then it may produce test tubes that are on average too large or too small.

Formulate the Null and Alternative Hypotheses to test whether the production process has gone out of adjustment.

*Example 11:*

A human resources manager is interested in testing the effectiveness of a new training program. Prior to the training, employees could handle an average of 11.65 cases per day. The HR manager would like to determine whether there is evidence that the training has increased the average number of cases employees can handle per work day.

Formulate the Null and Alternative Hypotheses to test whether the training increased the average number of cases employees can handle per day.